Arithmetic Average Transformation Technique to Solve Multi-Objective Quadratic Programming Problem

Najmaddin A. Sulaiman*, BasiyaK. Abulrahim*

[*] *Department of Mathematics, School of Science Education, Faculty of Education, University of Salahaddin, gardy.muhamad@yahoo.com.*

[**] *Department of Mathematics, School of Science Education, Faculty of Education, University of Garmian, basiya2008@yahoo.com.*

Abstract:

In this paper, we used arithmetic average transform technique for solving the multi-objective quadratic programming problem (MOQPP) to single-objective quadratic programming problem(SOQPP), through a new method using arithmetic average, then solve the problems by Wolfe's method [3,12].The obtain results are compared with that of modified method given in [11] and with the Chandra Sen. method [7].

Keywords: MOQPP, arithmetic average.

1. Introduction

The MOQP models are of greatest interest in our daily life. We are often concerned about the optimization of the product to linear factions like the summary cost of total transportation expenditures to the maximal necessary time to satisfy the demands, the total benefits or production value into time unit. A multi-objective linear programming problem (MOLPP) is solved by Chandra Sen. in (1983) [7]. In (1989) Sulaiman studied the computational aspects of single-objective in definite QPP [10]. In (1992) Mohammad and Sulaiman studied the MOF complimentary program [5]. In (1993) Abdil-Kadir and Sulaiman studied the multi-objective fractional programming problem (MOFPP) [1]. In (2006) Sulaiman and Sadiq studied the multi-objective function by solving the MOLPP, using mean and median value (Sadiq's approach). They did a comparison of results between Sen's approach and their modified approach [6]. In (2007) Sulaiman and Othman suggested an approach to construct the multi-objective function [8]. In (2008) Hamad-Amin [4] studied the multi-objective function by solving the MOLPP, using arithmetic average value. He did a comparison of results between Sen's approach, Sadiq's approach and arithmetic average. In (2010) Salih studied the multi-objective function by solving the MOLFPP, using mean, median and arithmetic average value. She did a comparison of results between Sen's approaches, mean, median and arithmetic average value [9]. In (2011) Sulaiman and Abdul-Rahim studied the optimal cutting procedure for MOQPP [11]. In order to extend this work we have defined a MOQPP and investigated an algorithm to solve QPP for multi-objective function, irrespective of number of objectives with less computational burden and suggest a new technique using arithmetic average values of objective functions, to generate the best optimal solution; also our algorithm has been discussed by numerical examples. Finally, result of the new different technique is presented and compared with Chandra Sen.'s, mean, and median techniques.

2. Mathematical Form of MOQPPA

MOLPP is introduced by Chandra Sen. in (1983)[7] and suggest an approach to construct the multi-objective function under limitation that the optimum value of individual problem is greater than zero, but he has not considered the situation when the optimum value of some of those functions may be negative or zero. The mathematical form of this type of problems is given as follows:

$$\left.\begin{array}{l} Max.\,Z_1 = C_1^T x + a_1 \\ Max.\,Z_2 = C_2^T x + a_2 \\ \quad . \\ \quad . \\ \quad . \\ Max.\,Z_r = C_r^T x + a_r \\ Min.\,Z_{r+1} = C_{r+1}^T x + a_{r+1} \\ \quad . \\ \quad . \\ \quad . \\ Min.\,Z_s = C_s^T x + a_s \end{array}\right\} \quad (1)$$

Subject to:

$$Ax \begin{bmatrix} \geq \\ \leq \\ = \end{bmatrix} b \quad (2)$$

$$x \geq 0 \quad (3)$$

Where r is number of objective functions to be maximized, s is the number of objective functions to be maximized and minimized, $(s-r)$ is the number of objective functionsthat is minimized,x is an n-dimensional vector of decision variables, $C$is the n-dimensional vector of constants, b is m-dimensional vector of constants, A is$(m \times n)$ matrix of coefficients. All vectors are assumed to be column vectors unless transposed,a_i are scalar constants, for each$i = 1,2, \dots, s$.Our aim is to generalize the algorithm suggested by Chandra Sen. [7] for solving MOLPP and we also present a technique for solving MOQPP. The mathematical form of this type of problems is given as follows:

$$\left.\begin{aligned} &Max.Z_1 = C_1^T x + \frac{1}{2}x^T G_1 x \\ &Max.Z_2 = C_2^T x + \frac{1}{2}x^T G_2 x \\ &\quad\vdots \\ &Max.Z_r = C_r^T x + \frac{1}{2}x^T G_r x \\ &Min.Z_{r+1} = C_{r+1}^T x + \frac{1}{2}x^T G_{r+1} x \\ &\quad\vdots \\ &Min.Z_s = C_s^T x + \frac{1}{2}x^T G_s x \end{aligned}\right\} \quad (4)$$

Subject to:

$$Ax \begin{bmatrix} \geq \\ \leq \\ = \end{bmatrix} b \quad (5)$$

$$x \geq o \quad (6)$$

Where G is$(n \times n)$matrix of coefficients with G is symmetric matrix.All vectors are assumed to be column vectors unless transposed $(T), i = 1,2, \dots, s$.

2.1 Formulation of MOQ Functions(MOQPP) The same approach has been taken by Abdil-kadir and Sulaiman[1] for MOF functions is followed here to formulate the constrained objective functions given in equation (4). Suppose that we have obtained a single value corresponding to each of the objective functions of it, being optimized individually subject to constraints(5) and (6) as follows:

$$\left.\begin{aligned} &Max.Z_1 = \varphi_1 \\ &Max.Z_2 = \varphi_2 \\ &\quad\vdots \\ [[&Max.Z_r = \varphi_r \\ &Min.Z_{r+1} = \varphi_{r+1} \\ &\quad\vdots \\ &Min.Z_s = \varphi_s \end{aligned}\right\} \quad (7)$$

Where $\varphi_1,\ \varphi_2, \dots,\ \varphi_r,\ \varphi_{r+1}, \dots, \varphi_s$, are the values of objective functions. The level of the decision variable may not necessary be common to all optimal solutions in presence of conflicts among objectives. But we required the common set of decision variable to be the best compromising optimal solution [2]. Hence we can determine the common set of decision variables from the following combined objective function[7]. Which formulate the MOQPP given in (4) as follows:

$$Max.Z = \sum_{k=1}^{r} \frac{Z_k}{|\varphi_k|} - \sum_{k=r+1}^{s} \frac{Z_k}{|\varphi_k|} \qquad (8)$$

Where $\varphi_k \neq 0$, for $k = 1,2,\ldots,r,r+1,\ldots,s$.Subject to the same constraints (5),(6) and the optimum value of the functions φ_k, $k = 1,2,\ldots,r,r+1,\ldots,s$ may be positive or negative (i.e.$\varphi_k \in R\backslash\{0\}$; where$R$ is the set of real numbers).

2.2 Numerical Examples

We use some numerical examples to solve MOQPP. **Example (1):** Solve the following MOQPP by using Chandra Sen.'s technique[7]

$$Max.Z_1 = -3x_1^2 - 3x_2^2 - 6x_1x_2 + 30x_1 + 30x_2 - 48$$
$$Max.Z_2 = -4x_1^2 - 4x_2^2 - 8x_1x_2 + 38x_1 + 38x_2 - 48$$
$$Max.Z_3 = -2x_1^2 - 2x_2^2 - 4x_1x_2 + 20x_1 + 20x_2 - 32$$
$$Max.Z_4 = -8x_1^2 - 8x_2^2 - 16x_1x_2 + 66x_1 + 66x_2 - 108$$
$$Max.Z_5 = -x_1^2 - x_2^2 - 2x_1x_2 + 10x_1 + 10x_2 - 16$$
$$Max.Z_6 = -3x_2^2 + 36x_1 + 34x_2$$
$$Max.Z_7 = -2x_1^2 + 32x_1 + 32x_2$$
$$Max.Z_8 = -3x_2^2 - 6x_1x_2 + 30x_1 + 30x_2$$
$$Min.Z_9 = 2x_1^2 + 2x_2^2 + 4x_1x_2 - 20x_1 - 20x_2 + 32$$
$$Min.Z_{10} = 5x_1^2 + 5x_2^2 + 10x_1x_2 - 42x_1 - 42x_2 + 34$$
$$Min.Z_{11} = 2x_1^2 + 2x_2^2 + 4x_1x_2 - 17x_1 - 17x_2 + 32$$
$$Min.Z_{12} = 6x_1^2 + 6x_2^2 + 12x_1x_2 - 50x_1 - 50x_2 + 90$$
$$Min.Z_{13} = 4x_1^2 - 36x_1 - 32x_2$$
$$Min.Z_{14} = 2x_2^2 - 34x_1 - 30x_2$$
$$Min.Z_{15} = 3x_2^2 + 6x_1x_2 - 32x_1 - 32x_2$$

Subject to:

$$x_1 + 2x_2 \leq 7$$
$$5x_1 + 2x_2 \leq 11$$
$$3x_1 + 2x_2 \leq 10$$
$$x_1,\ x_2 \geq 0$$

Solution(1): After finding the value of each of individual objective functions of example (1) by Wolfe's method [3, 12], the results obtained by using Chandra Sen.'s technique [7]are given and the numerical results as below in table 1:

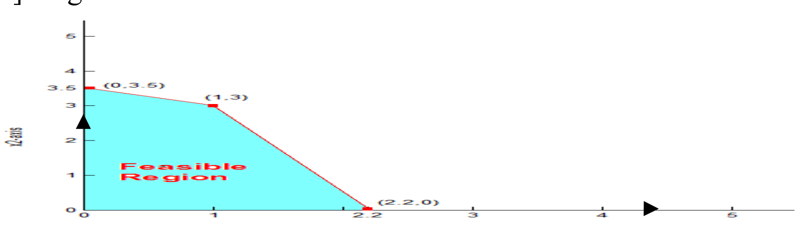

Figure 1: Feasible region for example (1)

Table1: Numerical results of example (1) for all objective functions

i	Z_i	x_i	φ_i	OA_i	OL_i
1	24	(1,3)	24	24	
2	40	(1,3)	40	40	
3	16	(1,3)	16	16	
4	28	(1,3)	28	28	
5	8	(1,3)	8	8	
6	111	(1,3)	111	111	
7	126	(1,3)	126	126	
8	75	(1,3)	75	75	
9	−16	(1,3)	−16		16
10	−54	(1,3)	−54		54
11	−4	(1,3)	−4		4
12	−14	(1,3)	−14		14
13	−128	(1,3)	−128		128
14	−106	(1,3)	−106		106
15	−83	(1,3)	−83		83

$$TG = \sum_{i=1}^{8} HG_i = -0.776587x_1^2 - 0.827741x_2^2 - 1.601429x_1x_2 + 8.035435x_1 + 8.017417x_2 - 11.057143$$

$$TL = \sum_{i=9}^{15} HL_i = 1.177414x_1^2 + 1.201177x_2^2 + 2.364617x_1x_2 - 10.836754x_1 - 10.767768x_2 + 17.058201$$

$$Max.Z = TG - TL = -1.954001x_1^2 - 2.028918x_2^2 - 3.966046x_1x_2 + 18.872189x_1 + 18.785185x_2 - 28.115344$$

Subject to:

$$x_1 + 2x_2 \le 7$$
$$5x_1 + 2x_2 \le 11$$
$$3x_1 + 2x_2 \le 10$$
$$x_1, x_2 \ge 0$$

After solving it by Wolfe's method [3, 12], we get $Max.Z = 14.999999$ and $x_1 = 1,\ x_2 = 3$

Example (2): Solve the following MOQPP by using Chandra Sen.'s technique [7]

$$Max.Z_1 = (4x_1 + 2x_2)(2x_1 + 2x_2 + 2)$$
$$Max.Z_2 = (5x_1 + x_2)(3x_1 + 3x_2 + 3)$$
$$Max.Z_3 = (4x_1 + 2x_2)(x_1 + x_2 + 1)$$
$$Max.Z_4 = (3x_1 + x_2)(3x_1 + 3x_2 + 3)$$
$$Min.Z_5 = (-2x_1 - x_2)(x_1 + x_2 + 1)$$
$$Min.Z_6 = (-5x_1 - 2x_2)(2x_1 + 2x_2 + 2)$$
$$Min.Z_7 = (-3x_1 - 2x_2)(x_1 + x_2 + 1)$$

Subject to:

$$x_1 + x_2 \geq 1$$
$$3x_1 + 2x_2 \leq 6$$
$$2x_1 + 4x_2 \leq 8$$
$$x_1,\ x_2 \geq 0$$

Solution(2): After finding the value of each of individual objective functions of example(2) by Wolfe's method [3, 12], the results obtained by using Chandra Sen.'s technique [7] are given and the numerical results as below in table 2:

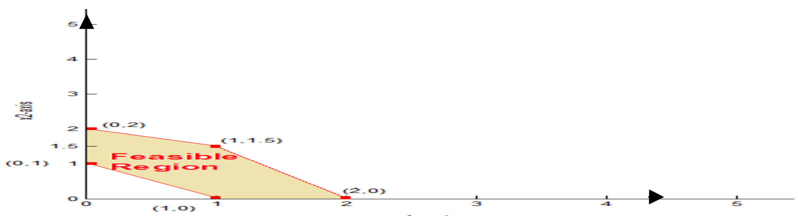

Figure 2: Feasible region for example (2)

Table 2: Numerical results of example (2) for all objective functions

i	Z_i	x_i	φ_i	OA_i	OL_i
1	48	(2,0)	48	48	
2	90	(2,0)	90	90	
3	24	(2,0)	24	24	
4	54	(2,0)	54	54	
5	−12	(2,0)	−12		12
6	−60	(2,0)	−60		60
7	−18	(2,0)	−18		18

$$TG = \sum_{i=1}^{4} HG_i = 0.666667x_1^2 + 0.255555x_2^2 + 0.922222x_1x_2 + 0.666667x_1 + 0.255555x_2$$

$$TL = \sum_{i=5}^{7} HL_i = -0.5x_1^2 - 0.261111x_2^2 - 0.761111x_1x_2 - 0.5x_1 - 0.261111x_2$$

$$Max. Z = TG - TL = 1.166667x_1^2 + 0.516666x_2^2 + 1.683333x_1x_2 + 1.166667x_1 + 0.516666x_2$$

$$Max. Z = (1.166667x_1 + 0.516666x_2)(x_1 + x_2 + 1)$$

Subject to:

$$x_1 + x_2 \geq 1$$
$$3x_1 + 2x_2 \leq 6$$
$$2x_1 + 4x_2 \leq 8$$
$$x_1, \quad x_2 \geq 0$$

After solving it by Wolfe's method [3, 12], we get

$$Max. Z = 7.000002 \text{ and } x_1 = 2,\ x_2 = 0$$

The solution for example (1) when applying algorithm in [11]using mean is the same optimal solution shown in table 1, then the combined objective quadratic function is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{8} Z_i = -20x_1^2 - 24x_2^2 - 42x_1x_2 + 262x_1 + 260x_2 - 252$$

$$VM = \frac{\sum_{i=1}^{r} OA_i}{r} = \frac{\sum_{i=1}^{8} OA_i}{8} = \frac{428}{8} = 53.5$$

$$S_1 = \frac{SM}{VM} = -0.373832x_1^2 - 0.448598x_2^2 - 0.785047x_1x_2 + 4.897196x_1 + 4.859813x_2 - 4.71028$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{9}^{15} Z_i = 19x_1^2 + 20x_2^2 + 36x_1x_2 - 231x_1 - 223x_2 + 188$$

$$VN = \frac{\sum_{i=r+1}^{s} OL_i}{s-r} = \frac{\sum_{9}^{15} OL_i}{7} = \frac{405}{7} = 57.857143$$

$$S_2 = \frac{SN}{VN} = 0.328395x_1^2 + 0.345679x_2^2 + 0.622222x_1x_2 - 3.992593x_1 - 3.854321x_2 + 3.249383$$

$$Max. Z = S_1 - S_2 = -0.702227x_1^2 - 0.794277x_2^2 - 1.407269x_1x_2 + 8.889789x_1 + 8.714134x_2 - 7.959663$$

After solving it by Wolfe's method [3, 12], it is subject to the same constraints as before we get$Max. Z = 15.000001$ and $x_1 = 1,\ x_2 = 3$

The solution for example(1) when applying algorithm in[11] by using median is the same optimal solution shown in Table 1, then the combined objective quadratic function is:

To find median,
First arrangeOA_ithat is $8, 16, 24, 28, 40, 75, 111, 126$ then $VM = 34$
And second arrangeOL_ithat is $4, 14, 16, 54, 83, 106, 128$ then $VN = 54$

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{8} Z_i = -20x_1^2 - 24x_2^2 - 42x_1x_2 + 262x_1 + 260x_2 -252$$

$$VM = \frac{40+28}{2} = \frac{68}{2} = 34$$

$$S_1 = \frac{SM}{VM} = -0.588235x_1^2 - 0.705882x_2^2 - 1.235294x_1x_2 + + 7.647059x_2 - 7.411765$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{9}^{15} Z_i = 19x_1^2 + 20x_2^2 + 36x_1x_2 - 231x_1 - 223x_2 +188$$

$$VN = 54$$

$$S_2 = \frac{SN}{VN} = 0.351852x_1^2 + 0.37037x_2^2 + 0.666667x_1x_2 - 4.277778x_1 - 4.12963x_2 + 3.481481$$

$$Max.Z = S_1 - S_2 = -0.940087x_1^2 - 1.076252x_2^2 - 1.901961x_1x_2 + 11.98366x_1 + 11.776689x_2 - 10.893246$$

After solving it by Wolfe's method [3, 12], it is subject to the same constraints as before we get $Max.Z = 20.088243$ and$x_1 = 1,\ x_2 = 3$

The solution for example (2) when applying algorithm in [11] by using mean is the same optimal solution shown in table 2, then the combined objective quadratic unction is:

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = 36x_1^2 + 12x_2^2 + 48x_1x_2 + 36x_1 + 12x_2$$

$$VM = \frac{\sum_{i=1}^{r} OA_i}{r} = \frac{\sum_{i=1}^{4} OA_i}{4} = \frac{216}{4} = 54$$

$$S_1 = \frac{SM}{VM} = 0.666667x_1^2 + 0.222222x_2^2 + 0.888889x_1x_2 + 0.666667x_1 +0.222222x_2$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{5}^{7} Z_i = -15x_1^2 - 7x_2^2 - 22x_1x_2 - 15x_1 - 7x_2$$

$$VN = \frac{\sum_{i=r+1}^{s} OL_i}{s-r} = \frac{\sum_{5}^{7} OL_i}{3} = \frac{90}{3} = 30$$

$$S_2 = \frac{SN}{VN} = -0.5x_1^2 - 0.233333x_2^2 - 0.733333x_1x_2 - 0.5x_1 - 0.233333x_2$$

$$Max.Z = S_1 - S_2 = 1.166667x_1^2 + 0.455555x_2^2 + 1.622222x_1x_2 + 1.166667x_1 + 0.455555x_2$$

$$Max.Z = (1.166667x_1 + 0.455555x_2)(x_1 + x_2 + 1)$$

After solving it by Wolfe's method [3, 12], it is subject to the same constraints as before we get $Max.Z = 7.000002$ and $x_1 = 2,\ x_2 = 0$

The solution for example(2) when applying algorithm in [11] by using median is the same optimal solution shown in table2, then the combined objective quadratic function is:

To find median,
First arrange OA_i that is 24, 48, 54, 90 then $VM = 51$
And second arrange OL_i that is 12, 18, 60 then $VN = 18$

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = 36x_1^2 + 12x_2^2 + 48x_1x_2 + 36x_1 + 12x_2$$

$$VM = \frac{48+54}{2} = \frac{102}{2} = 51$$

$$S_1 = \frac{SM}{VM} = 0.705882x_1^2 + 0.235294x_2^2 + 0.941176x_1x_2 + 0.705882x_1 + 0.235294x_2$$

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{5}^{7} Z_i = -15x_1^2 - 7x_2^2 - 22x_1x_2 - 15x_1 - 7x_2$$

$$VN = 18$$

$$S_2 = \frac{SN}{VN} = -0.833333x_1^2 - 0.388889x_2^2 - 1.222222x_1x_2 - 0.833333x_1 - 0.388889x_2$$

$$Max.Z = S_1 - S_2 = 1.539215x_1^2 + 0.624183x_2^2 + 2.163398x_1x_2 + 1.539215x_1 + 0.624183x_2$$

$$Max.Z = (1.539215x_1 + 0.624183x_2)(x_1 + x_2 + 1)$$

After solving it by Wolfe's method [3, 12], it is subject to the same constraints as before we get $Max.Z = 9.23529$ and $x_1 = 2,\ x_2 = 0$

3. Solving MOQPP by Arithmetic Average Technique

We formulate the combined objective function (8) as follows determine the common set of decision variables. To solve MOQPP by using arithmetic average technique (using arithmetic average value) consider below:

$$Max.Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i}{A_v} \tag{9}$$

Subject to the same constraints (5)and (6).

Where A_v denotes the arithmetic average, and evaluated as follows:

Let $m_1 = \min\{OA_i\}$, where $OA_i = |\varphi A_i|$, and φA_i is the maximum value of $Z_i, \forall i = 1,2,\dots,r$

Let $m_2 = \min\{OL_i\}$, where $OL_i = |\varphi L_i|$, and φL_i is the minimum value of Z_i, $\forall i = r+1, r+2,\dots,s$

$$A_v = \frac{m_1 + m_2}{2}$$

3.1 Algorithm of Arithmetic Average Technique

An algorithm for obtaining the optimal solution for the MOQPP defined in equation (4) which can be summarized as follows:

Step1: Assign arbitrary value to each of the individual objective functions which are to be maximized and minimized.

Step2: Solve the first objective function by the Wolfe's method [3, 12], for quadratic programming problem subjects to constraints.

Step3: Check the feasibility of the solution obtained in step2, if it is feasible go to step4, otherwise, use dual Wolfe's method to remove infeasibility.

Step4: Assign a name to the optimum value of the objective function Z_i .Say φ_i, for $i = 1,2,\dots,s$ as before.

Step5: Select $m_1 = \min\{OA_i\}$, $i = 1,2,..,r$
$m_2 = \min\{OL_i\}$, $i = r+1, r+2,..,s$
Then calculate $A_v = \frac{m_1+m_2}{2}$.

Step6: Construct the combined objective function which has the formula (9).

Step7: optimize the combined objective function under the same constraints (5) and (6).

3.2 The Arithmetic Average Technique

$$SM = \sum_{i=1}^{r} Z_i$$

$$SN = \sum_{i=r+1}^{s} Z_i$$

$$OA_i = |\varphi A_i|;\ \forall i = 1,2,\dots,r$$

$$OL_i = |\varphi L_i|;\ \forall i = r+1, r+2,\dots,s$$

$m_1 = \min\{OA_i\}$, where $OA_i = |\varphi A_i|$, and φA_i is the maximum value of Z_i, $\forall i = 1,2,\dots,r$

$Let\ m_2 = \min\{OL_i\}$, where $OL_i = |\varphi L_i|$, and φL_i is the minimum value of Z_i, $\forall i = r+1, r+2,\dots,s$

$$A_v = \frac{m_1 + m_2}{2}, \qquad Max.Z = \frac{(SM - SN)}{A_v}$$

Using arithmetic average technique to solve example (1) we get is the same optimal solution shown in table1, then the combined objective quadratic function is:

$$m_1 = \min\{OA_i\} = 8$$
$$m_2 = \min\{OL_i\} = 4$$
$$A_v = \frac{m_1 + m_2}{2} = \frac{8+4}{2} = \frac{12}{2} = 6$$
$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{8} Z_i = -20x_1^2 - 24x_2^2 - 42x_1x_2 + 262x_1 + 260x_2 -252$$
$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{9}^{15} Z_i = 19x_1^2 + 20x_2^2 + 36x_1x_2 - 231x_1 - 223x_2 +188$$
$$SM - SN = -39x_1^2 - 44x_2^2 - 78x_1x_2 + 493x_1 + 483x_2 - 440$$
$$Max.Z = \frac{SM - SN}{A_v} = -6.5x_1^2 - 7.333333x_2^2 - 13x_1x_2 + 82.166667x_1 + 80.5x_2 - 73.333333$$

After solving it by Wolfe's method [3, 12], it is subject to the same constraints we get $Max.Z = 138.833337$ and $x_1 = 1,\ x_2 = 3$

Using arithmetic average technique to solve example (2) we get is the same optimal solution shown in table 2, then the combined objective quadratic function is:

$$m_1 = \min\{OA_i\} = 24$$
$$m_2 = \min\{OL_i\} = 12$$
$$A_v = \frac{m_1 + m_2}{2} = \frac{24+12}{2} = \frac{36}{2} = 18$$
$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = 36x_1^2 + 12x_2^2 + 48x_1x_2 + 36x_1 + 12x_2$$
$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{5}^{7} Z_i = -15x_1^2 - 7x_2^2 - 22x_1x_2 - 15x_1 - 7x_2$$
$$SM - SN = 51x_1^2 + 19x_2^2 + 70x_1x_2 + 51x_1 + 19x_2$$
$$Max.Z = \frac{SM - SN}{A_v} = 2.833333x_1^2 + 1.055556x_2^2 + 3.888889x_1x_2 + 2.833333x_1 + 1.055556x_2$$

$$Max.Z = (2.833333x_1 + 1.055556x_2)(x_1 + x_2 + 1)$$

After solving it by Wolfe's method [3, 12], it is subject to the same constraints we get $Max.Z = 16.999998$ and $x_1 = 2,\ x_2 = 0$

4. Numerical Results Now, we are going to comparison the numerical results which are obtained of the examples as below in Table 3:

Table 3: Comparison between results of the numerical techniques

Techniques		O. S. of Example (1)	O.S. of Example (2)
Using Chandra Sen.'s Technique		$Max. Z = 14.999999$ $x_1 = 1$ $x_2 = 3$	$Max. Z = 7.000002$ $x_1 = 2$ $x_2 = 0$
Modified Technique	Using Mean	$Max. Z = 15.000001$ $x_1 = 1$ $x_2 = 3$	$Max. Z = 7.000002$ $x_1 = 2$ $x_2 = 0$
	Using Median	$Max. Z = 20.088243$ $x_1 = 1$ $x_2 = 3$	$Max. Z = 9.23529$ $x_1 = 2$ $x_2 = 0$
Using Arithmetic Average Technique		$Max. Z = 138.833337$ $x_1 = 1$ $x_2 = 3$	$Max. Z = 16.999998$ $x_1 = 2$ $x_2 = 0$

It is clear that the results obtained in examples(1),(2)when using arithmetic average technique are better than other results which are obtained by using Chandra Sen.; mean and median techniques.

5. Discussion

In this paper we have defined and discussed a number of techniques, which we have used in order to get the optimal solution of the MOQPP. The comparisons of these techniques are based on the values of the objective functions; therefore we have tested two MOQPP with linear constrained. To show the best technique among these methods we have obtained that arithmetic average technique was better than the techniques namely Chandra Sen.; mean and median techniques.

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پوختە

لەم لێكولینەوەیەدا، تەكنیكی گۆڕین بۆ شیكاركردنی كێشەكانی پرۆگرامی دووجایی فرەئامانج بە گۆڕینی بۆ كێشەكانی پرۆگرامی دووجایی تاك ئامانج بە دۆزینەوەی ڕێگایەكی نوێ بەبەكارهێنانی تێكڕایی ژمارەی گونجاو ئینجا شیكاركردنی كێشەكان بە ڕێگای وولف لە [12,3] **وەبەراوردكردنی ئەنجامەكان لەگەڵ ئەنجامەكانی دەستكەوتوو بەڕێگای پێشكەوتوو لە**[11] **و ڕێگای جاندراسین لە**[7].