Minimize of Error Estimation and Approximate Solution of Fifth Order Initial Value Problems

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Abstract:

In this paper, we derive some difference schemes for the initial value problems using approximate spline interpolation, with the sixth degree lacunary spline function that interpolates the solution function of the problem; first and fourth order accurate schemes are obtained. To test the efficiency of the method, several numerical examples of third and fifth order initial value problems are solved by proposed method.

Keywords: Differential equations, Spline interpolation, Absolute error, Lipchitz condition.

I. INTRODUCTION:

We consider the fifth order initial value problem

$$y^{(5)}(x) = f(x, y, y', y'', y''', y^{(4)}),\ x_0 \in [0, 1],$$
$$y^{(i)}(x_0) = y_{i+1},\ i = 0, 1, 2, 3, 4. \tag{1}$$

where $f \in C^6([0,1] \times R^5)$ is given and satisfies the (uniform) Lipchitz condition

$$\left| f^{(q)}(x, Y_1) - f^{(q)}(x, Y_2) \right| \le L \left| Y_1 - Y_2 \right|,$$
$$q = 0, 1, 2, 3, 4, 5. \tag{2}$$

Here
$Y_k = (y_k, y'_k, y''_k, y'''_k, y^{(4)}_k)$ where k = 0,1 and k indicated index of the vector Y_k and the norm Y is given by $\|Y\| = \sum_{i=0}^{4} \left| y^{(i)} \right|$ and L is the Lipchitz constant.
We are given the mesh points:

$$\Delta : 0 = x_0 < x_1 < \cdots < x_n = 1$$

with $x_{k+1} - x_k = h,\ k = 0, 1, \cdots, n-1$, and n be a uniform partition numbers of the interval $[0, 1]$, with these conditions the problem (1) is well posed [2, 3].

Problem (1) models many practical problems in various areas of applied mathematics and physics. There are some principal approaches to numerically solve the model problems like (1), namely, the finite difference method, the finite element method and the spline approximation methods in [11].

Meir and Sharma [7] have initiated the study of the lacunary interpolation and its applicability to approximate solutions of differential equations. The area of the present paper is concerned with the spline approximation method, to solve the problems of type (1). We use the lacunary spline interpolation functions of type (0, 1, 4) [4] to approximate problem (1). Also the Cauchy problem has been used by many authors for solving these problems. Gyovari solved the Cauchy problem by using a modified lacunary spline function which interpolates the lacunary data (0, 2, 3) [2, 3]. Saxena used deficient lacunary spline for solving the Cauchy problem [10]; Rostam, Faraidun and Gulnar [8] discussed ninth degree spline method for solving the system of ordinary differential equations also, and also see [1, 5 and 9] and the references there in.

In this paper we have shown that by making use of continuity of the first and fourth derivatives of the spline function, the result of two spline difference schemes gives a new type that can be solved efficiently by the well-known algorithm to the fourth order initial value problem.

II. Description of the method

To solve the fifth order initial value problems (1), we define approximation for $\bar{S}_\Delta(x)$:

Let $x_k = \frac{k}{n};\ k = 0,1,\ \ldots,\ n$, $h = \frac{1}{n}$, $\omega(h,\ y^{(r)}) = \underset{|x-x_1|\le h}{Max}\left\{\left|y^{(r)}(x) - \bar{y}^{(r)}(x)\right|\right\}$, $r = 0, 1, \cdots, 6$ and let $\bar{Y}_k^{(q)} : \bar{y}_0^{(q)},\ \bar{y}_1^{(q)},\ \bar{y}_2^{(q)},\ \cdots,\ \bar{y}_n^{(q)}$, $q = 0, 1, \cdots, 6$, be approximations to the exact values $Y_k^{(q)} : y_0^{(q)},\ y_1^{(q)},\ y_2^{(q)},\ \cdots, y_n^{(q)}$. Now from these approximate values we construct a spline function $\bar{S}_\Delta(x)$ which interpolates to the set $\bar{Y}$ on the mesh Δ and approximates the solution $y(x)$ of (1). The set $\bar{Y}^{(q)}$ is defined as:

$$\bar{y}_0 = y_0,\ \bar{y}_0'' = y_0'',\ \bar{y}_0^{(2+q)} = f^{(q)}(x_0, y_0, y_0''),\ q = 0, 1, \cdots, r,$$

$$\bar{y}_{k+1} = \bar{y}_k + h\bar{y}'_k + \int_{x_k}^{x_{k+1}} \int_{x_k}^{t} f\ [u, y_k^*(u), y_k^{**}{}'(u)]du\, dt\ ,$$

$$\bar{y}'_{k+1} = \bar{y}'_k + \int_{x_k}^{x_{k+1}} f\ [t,\ y_k^*(t),\ y_k^{**}{}'(t)]dt\ ,$$

$$\bar{y}_{k+1}^{(q+2)} = f^{(q)}(x_{k+1}, \bar{y}_{k+1}, \bar{y}'_{k+1}),\quad q = 0, 1, 4,\quad k = 0, 1, 2, \cdots, m-1$$

and for $x_k \le x \le x_{k+1}$,

$$y_k^*(x) = \sum_{j=0}^{r+2} (x - x_k)^j \frac{\bar{y}_k^{(j)}}{j!},\ y_k^*{}'(x) = \sum_{j=0}^{r+1} (x - x_k)^j \frac{\bar{y}_k^{(j+1)}}{j!}$$

and

$$y'^{**}_{k+1}(x) = \bar{y}'_k + \int_{x_k}^{x_1} f\ [t,\ y_k^*(t),\ y_k^*{}'(t)]dt\ .$$

We use the lacunary spline interpolation function of the type (0, 1, 4) of [4] to approximate problem (1). Also, $\bar{S}_\Delta(x) = \bar{S}_k(x)$ if $x_k \le x \le x_{k+1}$ and denote by $\bar{S}_{n,6}^5$ the class of six degree splines $\bar{S}(x)$ as

$$G(x) = \begin{cases} \bar{S}_\Delta(x_k) = \bar{y}_k \\ \bar{S}_\Delta^{(q)}(x_k) = \bar{y}_k^{(q)}, \end{cases} \tag{3}$$

where $q = 1,4$ and $k = 0,1, 2,\cdots,m$, the existence and uniqueness of the above spline function have been shown in [4].

$$\bar{S}_0 = \bar{y}_0 + (x - x_0)\bar{y}_0' + \frac{(x - x_0)^2}{2}\bar{y}_0'' + (x - x_0)^3\bar{a}_{0,3} + \frac{(x - x_0)^4}{24}\bar{y}_o^{(4)} + (x - x_0)^5\bar{a}_{0,5} + (x - x_0)^6\bar{a}_{0,6} \tag{4}$$

Let us now examine the intervals $[x_i, x_{i+1}]$, $i = 1,2,\cdots,n-2$, and define $\bar{S}_i(x)$ as

$$\bar{S}_i(x)=\bar{y}_i+(x-x_i)y_i'+(x-x_i)^2\bar{a}_{i,2}+(x-x_i)^3\bar{a}_{i,3}+\frac{(x-x_i)^4}{24}\bar{y}_i^{(4)}+(x-x_i)^5\bar{a}_{i,5}+(x-x_i)^6\bar{a}_{i,6}. \quad (5)$$

From [4], we can find the following coefficients by using the equation (3) as:

$$\bar{a}_{0,3}=\frac{3}{h^3}(\bar{y}_1-\bar{y}_0)-\frac{1}{3h^2}(2\bar{y}_1'+7\bar{y}_0')-\frac{5}{6}h\ \bar{y}_0''+\frac{h^4}{360}(\bar{y}_1^{(4)}-6\bar{y}_0^{(4)}),$$

$$\bar{a}_{0,5}=\frac{3}{h^5}(\bar{y}_0-\bar{y}_1)+\frac{1}{h^4}(\bar{y}_1'+2\bar{y}_0')+\frac{1}{2h^3}\bar{y}_0'-\frac{1}{120h}(\bar{y}_1^{(4)}+4\bar{y}_0^{(4)})$$

and

$$\bar{a}_{0,6}=\frac{1}{h^6}(\bar{y}_1-\bar{y}_0)-\frac{1}{3h^5}(\bar{y}_1''+2\bar{y}_0'')-\frac{1}{6h^4}\bar{y}_0''+\frac{1}{360h^2}(2\bar{y}_1^{(4)}+3\bar{y}_0^{(4)}).$$

also

$$\bar{a}_{i,2}-\bar{a}_{i+1,2}=\frac{6}{h^2}(\bar{y}_{i+1}-\bar{y}_i)+\frac{3}{h}(\bar{y}_{i+1}''-\bar{y}''_i)-\frac{h^2}{120}(\bar{y}_{i+1}^{(4)}-\bar{y}_i^{(4)}),$$

$$\bar{a}_{i,3}=-\frac{5}{3}h^{-1}\bar{a}_{i+1,2}-\frac{7}{h^3}(\bar{y}_{i+1}-\bar{y}_i)+\frac{1}{3h^2}(13\bar{y}_{i+1}''+8\bar{y}_i'')+\frac{h}{360}(6\bar{y}_{i+1}^{(4)}-11\bar{y}_i^{(4)}),$$

$$\bar{a}_{i,5}=\frac{1}{h^3}\bar{a}_{i+1,2}+\frac{3}{h^5}(\bar{y}_{i+1}-\bar{y}_i)-\frac{1}{h^4}(2\bar{y}_{i+1}''+\bar{y}_i'')-\frac{1}{120h}(3\bar{y}_{i+1}^{(4)}+2\bar{y}_i^{(4)})$$

and

$$\bar{a}_{i,6}=-\frac{1}{3h^4}\bar{a}_{i,2}-\frac{1}{h^6}(\bar{y}_{i+1}-\bar{y}_i)+\frac{1}{3h^5}(2\bar{y}_{i+1}''+\bar{y}_i'')+\frac{1}{360h^2}(3\bar{y}_{i+1}^{(4)}+2\bar{y}_i^{(4)}).$$

Similarly for the last interval $[x_n,x_{n-1}]$, we can putting it for $\bar{S}_n(x)$ in the end of the interval.

III. Theorem of Convergence

In this section, we find the analysis convergence for the new approximate spline function $\bar{S}_\Delta(x)$ given in the previous section to the exact solution of the fourth order initial value problem (1) corresponding to the values of y_k ($k=0,1,2,....,n$) of problem (1), and prove the following theorem.

Theorem 1.

Let $\bar{y}_k^{(q)}$ $(q=0,1,4;k=0,1,2...,n)$ be the approximate values defined before. Then the following estimates of the spline function $\bar{S}_\Delta(x)$ are valid:

(i) $\left|S_k^{(q)}(x)-\bar{S}_k^{(q)}(x)\right|\le C\,h^{6-q}\,\omega_6(h)$; for $q=0,1,....,6,\ k=0,1,...,n-2$ where C_k denote the constants dependent of h, and $\omega_6(h)=\omega(h,y^{(6)})$.

(ii) $\left|y_k^{(q)}(x)-\bar{S}_k^{(q)}(x)\right|\le T_k^*h^{6-q}\,\omega_6(h)$; for $q=0,1,....,6$, where $y(x)$ is the exact solution of problem (1) and T_k^* denote the difference constants dependent of h, and $\omega_6(h)=\omega(h,y^{(6)})$.

Proof. (i) From theorem 1 of [4] and (3), we have

$$S_0(x)-\bar{S}_0(x)=(x-x_0)^3(a_{0,3}-\bar{a}_{0,3})+(x-x_0)^5(a_{0,5}-\bar{a}_{0,5})+(x-x_0)^6(a_{0,6}-\bar{a}_{0,6}), \quad (6)$$

where

$$a_{0,3}-\bar{a}_{0,3}=\frac{3}{h^3}(y_1-\bar{y}_1)-\frac{2}{3h^2}[y_1'-\bar{y}_1']+\frac{h}{360}[y_1^{(4)}-\bar{y}_1^{(4)}]$$

implies that

$$\left|a_{0,3}-\bar{a}_{0,3}\right|\le\frac{1}{360}(C_1+240C_2+1080C_3)\,\omega_6(h)=\frac{1}{360}T_1\,\omega_6(h),$$

where $T_1=C_1+240C_2+1080C_3$ and C_1,C_2 and C_3 are constants dependent of *h*. Similarly

$$\left|a_{0,5}-\bar{a}_{0,5}\right|\le\frac{3}{h^5}\left|y_1-\bar{y}_1\right|+\frac{1}{h^4}\left|y_1'-\bar{y}_1'\right|+\frac{h}{120}\left|y_1^{(4)}-\bar{y}_1^{(4)}\right|$$

$$\le\frac{1}{120}(C_4+120C_5+360C_6)\omega_6(h)=\frac{1}{120}T_2\omega_6(h)$$

where $T_2=C_4+120C_5+3600C_6$ and C_4,C_5 and C_6 are constants dependent of *h*.

$$\left|a_{0,6}-\bar{a}_{0,6}\right|\le\frac{1}{h^6}\left|y_1-\bar{y}_1\right|+\frac{1}{3h^5}\left|y_1'-\bar{y}_1'\right|+\frac{1}{180h^2}\left|y_1^{(4)}-\bar{y}_1^{(4)}\right|$$

$$\le\frac{1}{180}(C_7+60C_8+180C_9)\omega_6(h)=\frac{1}{180}T_3\omega_6(h),$$

where $T_3=C_7+60C_8+180C_9$ and C_7,C_8 and C_9 are constants dependent of *h* and hence

$$\left|S_0(x)-\bar{S}_0(x)\right|\le h^3\left|a_{0,3}-\bar{a}_{0,3}\right|+h^5\left|a_{0,5}-\bar{a}_{0,5}\right|+h^6\left|a_{0,6}-\bar{a}_{0,6}\right|$$

$$\le C\omega_6(h)$$

where $C=T_1+T_2+T_3$ dependent of *h*.

By taking the first derivative of (5), we have $\left|S_0'(x)-\bar{S}_0'(x)\right|=y_1-\bar{y}_1$, and from equation (3), we obtain:

$$\left|S_0''(x)-\bar{S}_0''(x)\right|\le\frac{12}{h^2}\left|\bar{y}_1-y_1\right|+\frac{6}{h}\left|y_1'-\bar{y}_1'\right|+\frac{h^3}{60}\left|y_1^{(4)}-\bar{y}_1^{(4)}\right|$$

$$\le\frac{1}{60}(\bar{C}_1+360\bar{C}_2+720\bar{C}_3)\,\omega_6(h)=\frac{1}{60}\bar{T}_1\omega_6(h),$$

and by successive differentiations we obtain

$$\left|S_0^{(q)}(x)-\bar{S}_0^{(q)}(x)\right|\le T_q\,h^{6-q}\,\omega_6(h),\quad q=0,1,\cdots,6.$$

This proves (i) for $k=0$ and $x\in[x_0,x_1]$. Furthermore, in the interval $[x_{k-1},x_k]$

$$S_k(x)-\bar{S}_k(x)=(x-x_k)^2(a_{k,2}-\bar{a}_{k,2})+(x-x_k)^3(a_{k,3}-\bar{a}_{k,3})+(x-x_k)^5(a_{k,5}-\bar{a}_{k,5})^5+(x-x_k)^6(a_{k,6}-\bar{a}_{k,6}).$$

From Jwamer and Kareem (2010), it is clear that

$$a_{k,2}-\bar{a}_{k,2}=a_{k+1,2}-\bar{a}_{k+1,2}+\frac{6}{h^2}(y_{i+1}-\bar{y}_{i+1})-\frac{3}{h}(y_{i+1}'-\bar{y}_{i+1}')+\frac{h^2}{120}(y_1^{(4)}-\bar{y}_1^{(4)}),$$

which implies that

$$\left|a_{k,2}-\bar{a}_{k,2}\right|\le\frac{1}{120}(C_0^*+720C_1^*+360C_2^*+120C_3^*)\,\omega_6(h)=\frac{1}{360}T_1^*\,\omega_6(h),$$

where T_1^* and C_0^*,C_1^*,C_2^* and C_3^* be constants dependent of *h*.

Similarly,

$$\left|a_{k,3}-\bar{a}_{k,3}\right| \le \frac{5}{3h}\left|\bar{a}_{k+1,2}-a_{k+1,2}\right|+\frac{7}{h^3}\left|y_{k+1}-\bar{y}_{k+1}\right|+\frac{13}{3h^2}\left|\bar{y}'_{k+1}-y'_{k+1}\right|+\frac{h}{60}\left|y^{(4)}_{k+1}-\bar{y}^{(4)}_{k+1}\right|$$

$$\le \frac{1}{60}(100C_3^*+420C_4^*+260C_5^*+C_6^*)\ \omega_6(h)=\frac{1}{60}T_2^*\ \omega_6(h),$$

where T_1^* and C_3^*, C_4^*, C_5^* and C_6^* be constants dependent of h. And also

$$\left|a_{k,5}-\bar{a}_{k,5}\right| \le T_3^*\ \omega_6(h)\ ;\ \left|a_{k,6}-\bar{a}_{k,6}\right| \le T_4^*\ \omega_6(h),$$

where T_2^* and T_4^* are dependent of h.

By taking successive differentiations, we obtain $\left|S_k^{(q)}(x)-\bar{S}_k^{(q)}(x)\right| \le T_q\, h^{6-q}\ \omega_6(h)$ for $q=0,1,\cdots,6$. which proves (i) for $k=0,1,\cdots,m-2.$ We can repeat the same manner in above for $k=m-1$.

Proof of Theorem 1 (ii).

$$\left|y^{(q)}(x)-\bar{S}_\Delta^{(q)}(x)\right| \le \left|y^{(q)}(x)-S_\Delta^{(q)}(x)\right|+\left|S_\Delta^{(q)}(x)-\bar{S}_\Delta^{(q)}(x)\right|$$

From theorem 2 of Kells [6], the following estimates are valid

$$\left|y^{(q)}(x)-\bar{S}_\Delta^{(q)}(x)\right| \le C_q\, h^{6-q}\ \omega_6(h). \tag{7}$$

Using (7) and the estimate in (i), we have

$$\left|y^{(q)}(x)-\bar{S}_\Delta^{(q)}(x)\right| \le C_q\, h^{6-q}\ \omega_6(h)+T_q\, h^{6-q}\ \omega_6(h)$$

$$=(C_q+T_q)\, h^{6-q}\ \omega_6(h)$$

$$=D_q\, h^{6-q}\ \omega_6(h),$$

which proves (ii).

Theorem 2.

If the function f in problem (1) satisfies conditions (2) and (3), then the following inequalities hold:

$\left|\bar{S}''_0(x)-f\,[x,\bar{S}_0(x),\bar{S}'_0(x)]\right| \le T_{0,2}^*\ \omega_6(h)$ where $T_{0,2}^*$ is a constant dependent of h and $x\in[x_0,x_1]$,

$\left|\bar{S}''_k(x)-f\,[x,\bar{S}_k(x),\bar{S}'_k(x)]\right| \le T_{k,2}^*\ \omega_6(h)$ where $T_{k,2}^*$ are constants dependent of h and $x\in[x_{k-1},x_k]$,

$\left|\bar{S}''_{m-1}(x)-f\,[x,\bar{S}_{m-1}(x),\bar{S}'_{m-1}(x)]\right| \le T_{m-1,2}^*\ \omega_6(h)$ where $T_{m-1,2}^*$ are constants dependent of h and $x\in[x_{m-1},x_m]$.

Proof. Using conditions (2) and (3), we have

$$\left|\bar{S}''_\Delta(x)-f\,[x,\bar{S}_\Delta(x),\bar{S}'_\Delta(x)]\right| \le \left|\bar{S}''_\Delta(x)+y''(x)-y''(x)-f\,[x,\bar{S}_\Delta(x),\bar{S}'_\Delta(x)]\right|$$

$$\le \left|\bar{S}''_\Delta(x)-y''(x)\right|+\left|y''(x)-f\,[x,\bar{S}_\Delta(x),\bar{S}'_\Delta(x)]\right|$$

$$\le \left|\bar{S}''_\Delta(x)-y''(x)\right|+L\left\{\left|\bar{S}_\Delta(x)-y(x)\right|+\left|\bar{S}'_\Delta(x)-y'(x)\right|\right\}.$$

This proves Theorem 2 with the help of Theorem 1.

In a similar manner, Theorem 2 was proved under different conditions by [10] and also by [2, 3].

Technical Algorithm:

Step 1: Partition $[a,b]$ into N subintervals I.

Step 2: Set

$\bar{S}_i' = \bar{y}_i'$ (i=0, 1, 2, …, N), $\bar{S}_i = \bar{y}_i$ (i=0, 1, 2, …, N)

$\bar{S}_i^{(4)} = \bar{y}_i^{(4)}$ (i=0, 1, 2, …, N-1)

and with initial condition $\bar{S}_0''' = \bar{y}_0'''$.

Step 3: Use (Theorem 1 (i)) to find $S_i - \bar{S}_i$, $i = 1, 2, \cdots, N$.

Step 4: Use (Theorem 1 (i)) to find the derivatives of $S_i - \bar{S}_i$ at N equally spaced points in each subinterval $x \in [x_{i-1}, x_i]$ go to step 5, else i=i+1 and repeat this iteration to find a proper i.

Step 5: Stop.

IV. Numerical results

We proceed to show numerical tests of the described algorithms for the first time. Also we analyze the local regularity of the derivative method and we consider two numerical examples illustrating the comparative performance of the spline method. All calculations are implemented by Matlab program [11]. For the sake of comparison we also tabulate the results (see Table 1 and 2) showing that the present method for the error bounds is obtained for a small step size h.

Problem 1. [6] Consider a fifth order initial value problem

$$y^{(5)} - y^{(4)} - y' + y = 0 \text{ where } x \in [0,1],$$

$$y(0) = y'(0) = y'''(0) = y^{(4)}(0) = 0 \text{ and } y''(0) = 1,$$

which has the exact solution $y(x) = \frac{1}{4}e^{-x} + \frac{1}{4}e^{x} - \frac{1}{2}\cos(x)$.

Table 1. Absolute maximum error for the derivatives $\bar{S}(x)$

h	$\lVert \bar{s}''(x) - y''(x) \rVert_\infty$	$\lVert \bar{s}'''(x) - y'''(x) \rVert_\infty$	$\lVert \bar{s}^{(5)}(x) - y^{(5)}(x) \rVert_\infty$	$\lVert \bar{s}^{(6)}(x) - y^{(6)}(x) \rVert_\infty$
0.5	1.6793×10^{-7}	5.2575×10^{-6}	1.2298×10^{-4}	3.1892×10^{-4}
0.1	4.0717×10^{-13}	6.8215×10^{-11}	3.8569×10^{-8}	4.9361×10^{-7}
0.05	1.5892×10^{-14}	2.4074×10^{-13}	3.9×10^{-9}	1.3864×10^{-7}
0.01	7.4797×10^{-12}	2.6031×10^{-9}	2.2319×10^{-4}	4.46×10^{-2}

Problem 2. Consider a third order initial value problem

$$y''' - y' = 2\cos(t) \text{ where } t \in [0,1],$$

$$y(0) = 3, y'(0) = 2 \text{ and } y''(0) = 1,$$

which has the exact solution $y(x) = 2 + 2e^{t} - e^{-t} - \sin(t)$.

Table 2. Absolute maximum error for the derivatives $\bar{s}(x)$

h	$\lVert\bar{s}''(x)-y''(x)\rVert_\infty$	$\lVert\bar{s}'''(x)-y'''(x)\rVert_\infty$	$\lVert\bar{s}^{(5)}(x)-y^{(5)}(x)\rVert_\infty$	$\lVert\bar{s}^{(6)}(x)-y^{(6)}(x)\rVert_\infty$
0.5	7.6×10^{-3}	5.29×10^{-2}	5.784×10^{-1}	10.056×10^{-1}
0.1	1.4458×10^{-5}	5.7459×10^{-4}	18.37×10^{-2}	18.081×10^{-1}
0.05	9.2060×10^{-7}	7.4114×10^{-5}	9.59×10^{-2}	19.044×10^{-1}
0.01	1.5349×10^{-9}	5.93141×10^{-7}	2.11×10^{-2}	22.264×10^{-1}

V. Conclusion

The computations associated with the two examples discussed above were performed using Matlab programming for the new algorithm. Tables 1 and 2 show that the accuracy of the spline method with respect to the variety of order is better than the results of [4] from the minimization of error bounds higher order derivatives, moreover, the existence and uniqueness of the solution is guaranteed theoretically. This method is clearly reliable if compared with grid point techniques where the solution is defined at grid points only. Moreover the method yields a good result even for small size *h*. As it can seen in the above tables, our presented method provides encouraging results and finding the solution of further higher order initial value problems also yields a good minimization error bounds.

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کەم کردنەوەی هەڵەی خەمڵێنراو لەگەڵ شیکاری نزیکراوەیی بۆ پلە پێنجی کێشەی نرخە سەرەتاییەکان

پوختە

لەم توێژینەوە دا، توانرا جۆرێکی نوێی سپلاَین پلە شەش بەدەست بهێنین بۆ هەندێ لە جۆرەکانی هاوکێشەی جیاکاری نرخە سەرەتاییەکان، وە بەدەست هێنانی سپلاینی نزیکراوەیی بۆ شیکاری کێشەکان بۆ بۆشلیەکانی یەکەم و چوارەم کە دەستمان کەوت. بۆ ئە نجام دانی توانای ئەم ڕێگایە، چەند نمونەیەکی ژمارەیمان لە پلەی سێیەم و پێنجەم شیکار کرا. کەبە تەواوی پشتگیری ئە نجامی تیۆریەکەی دەکرد.