A Generalization of Epiform Modules

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Received: 17 Jan. 2014,Revised: 7 Feb. 2014, Accepted : 24 Feb. 2014
Published online: 26 Mar. 2014

Abstract

In this paper we introduce a generalization of epiform modules . A module M is called an $f-epiform$ **module if every proper finitely generated submodule of M is a corational submodule of M.Also we investigate some basic properties of this type of modules. We studied some relationships between this type of modules and some other modules**.

Keywords: Corational submodules ,Epiform modules , Semihollw modules ,Hollow modules.

1. Introduction

Throughout this paper R is an associative ring with unity and all R-modules are left until.

$A \le M$ will mean A is a submodule of M .
Let M be a module and A be a submodule of M, A is called a small submodule of M (denoted by $A << M$) if for any $X \le M$, $M = X + A$ implies $X = M$.
A module M is called a hollow module if every proper submodule of M is a small submodule of M .
A module M is called a semihollow module if every proper finitely generated submodule is small submodule of M . It is known that every hollow module is semihollow .Recall that a module is called a local if it has a unique maximal submodule which contains all proper submodules of M [10].
A submodule A of a module M is called a corational submodule in M denoted by ($A \subseteq_{cr} M$), if $\mathrm{Hom}(M, \frac{N}{K}) = 0$ for all submodule K of N .A module M is called epiform if every proper submodule of M is corational submodule of M [12]. Equivalently, a module M is called epiform module if every nonzero homomorphism $f : M \to \frac{M}{K}$ with K a proper submodule of M is epimorphism.Every simple module is an epiform module.A module M is called almost finitely generated if M is not finitely generated and every proper submodule of M is finitely generated . Every almost finitely generated module is an epiform module. Every corational submodule is small and implies that every epiform module is hollow.If every small submodule of a module M is corational then this module is called a copolyform module.A module M is called an epiform module if every proper submodule of M is a coational submodule of M [12] .

Examples

1-The Z -module Z_{p^∞} is an epiform module.

2-Simple modules are epiform modules.

2. Basic Properties of $f-epiform$ Modules

Proposition 2.1. An epimorfic image of an f-epiform module is $f-epiform$.

Proof. Let M be an $f-epiform$ module and $f:M\to M'$ be an epimorphism.To show M' is also $f-epiform$, let X be a proper finitely generated submodule of M', it is enough to show that $X\subseteq_{cr} M'$ which means that $\mathrm{Hom}(M',\frac{X}{N})=0$ for all $N\le X$ that is if we take any homomorphism $h:M'\to\frac{X}{N}$ N then $h=0$ .Now $f^{-1}(X)$ is a proper finitely generated submodule of M this implies that $f^{-1}(X)\subseteq_{cr} M$ since M is a $f-epiform$ module. Hence for each homomorphism $g:M\to\frac{f^{-1}(X)}{f^{-1}(N)}$ we have $g(M)=f^{-1}(N)$ so that $f(g(M))=f(f^{-1}(N))=N$, since f is an epimorphism.Thus $(fog)(M)=N$ and one can easily we show that $(fog)=(hof)$ hence $h=0$.

Corollary 2.2. Let M be an $f-epiform$ module then $\frac{M}{N}$ is an $f-epiform$ module for all proper submdule N of M
.

Proof. Let M be an $f-epiform$ module and $\pi:M\to\frac{M}{N}$ be a natural epimorphism then by (prop.2.1) $\pi(M)=\frac{M}{N}$ is also an $f-epiform$ module.

Proposition 2.3. Let M be a Noetherian module then M is an $f-epiform$ module if and only if it is epiform.

Proof.
As M is Noetherian, then every sbmodule of M is finitely generated if M is $f-epiform$ then every submodule is a corational submodule and hence it is epiform. Conversely is clear.

Proposition 2.4. Every $f-epiform$ module is semihollow.

Proof. Let U be a proper finitely generated submodule of M then U is a corational submodule and every corational submodule is small then $U<<M$ and hence every proper finitely generated submodule of M is a small submodule of M .Thus M is a semihollow module.

Proposition 2.5. Every epiform module is an $f-epiform$ module

Proof. Clear.

Proposition 2.6. A direct summand of an $f-epiform$ module is $f-epiform$.

Proof.
Let $M=M_1\oplus M_2$ be an $f-epiform$ module where M_1 and M_2 are submodules of M . Since M is an $f-epiform$ module then by (corollary 2.2) $\frac{M}{M_1}$ is an $f-epiform$ module .Also we have $\frac{M}{M_1}\cong M_2$ and $\frac{M}{M1}$ $f-epiform$ thus M_2 is an $f-epiform$ module.

We have every $f-epiform$ module is a semihollow module then it is easy to verify that:

1- Every $f-epiform$ module is f-supplemented.
2- Every $f-epiform$ module over an Artinian ring is a Noetherian module.

3- Let M be an $f-epiform$ module and $RadM$ is a Noetherian module then M is a Noetherian module.

4- Let M be an $f-epiform$ module and $RadM$ is an Artinian module then M is an Artinian module.

5- A simple module over a field is the only $f-epiform$ module.

6- Let M be a module with $RadM \neq M$,if M is an $f-epiform$ module then M is local.

7- Let M be a finitely generated module ,if M is $f-epiform$ then it is local.

8-Let $f : P \to M$ be a projective cover of M, then M is $f-epiform$ if and only if P is $f-epiform$.

Proposition2.7. Let M be a module over a $V-ring$ R, if M is $f-epiform$ then M is a hollow(simple) module.

Proof. Let M be a module over a $V-ring$ $V-ring$ R which is $f-epiform$ then M is a semihollow module.By [10],we have a module over $V-ring$ is semihollow if and only if it is hollow thus M is a hollow(simple) module.

Proposition2.8. Let M be a module over a $V-ring$ R , if M is epiform then M semihollow.

Proof.similar to 4.1.

Proposition2.9. Let M be a module over a perfect (max) ring R if M is $f-epiform$ then M is a hollow(local) module

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